$a)$ *Exprese en forma rectangular*:

$$\left(\frac{2+i}{1-i}+\frac{2-i}{1+i}+i\right)^2$$

$$=\left(\frac{(2+i)(1+i)+(2-i)(1-i)}{(1-i)(1+i)}+i\right)^2=$$

$$\left(\frac{2+2i+i+i^2+2-2i-i+i^2}{2}+i\right)^2=\left(\frac{4-2}{2}+i\right)^2=(1+i)^2$$

$$=1+2i+i^2=1+2i-1=\boxed{\boldsymbol{2i}}$$

$b)$ *sea* $z_1 = 2 + 3i\ ; z_2 = -5i\ ; z_3 = 4 + 2i\ ; z_4 = 5 - 2i$; *Exprese en forma Polar*:

$$1)\ z_1 z_2 z_3$$

$$= (2 + 3i)(-5i)(4 + 2i) = (-10i - 15i^2)(4 + 2i) = (15 - 10i)(4 + 2i)$$

$$60 + 30i - 40i - 20i^2 = 60 - 10i + 20 = \boxed{80 - 10i}$$

$$r = \sqrt{(80)^2 + (-10)^2} = \boxed{10\sqrt{65}}$$

$$\theta = \arctan\left(\frac{-10}{80}\right) = -0.124 \therefore \theta = 6.158$$

$$luego\ z_1 z_2 z_3 = \boxed{\mathbf{10\sqrt{65}(\cos(6.158) + \sin(6.158)\, i)}}$$

$$2)\frac{z_1}{z_4}$$

$$=\frac{2+3i}{5-2i}\cdot\frac{5+2i}{5+2i}=\frac{(2+3i)(5+2i)}{25+4}=\frac{10+4i+15i+6i^2}{29}$$

$$\frac{10+19i-6}{29}=\boxed{\frac{4}{29}+\frac{19}{29}i}$$

$$r=\sqrt{\left(\frac{4}{29}\right)^2+\left(\frac{19}{29}\right)^2}=\boxed{\frac{\sqrt{377}}{29}}$$

$$\theta=\arctan\left(\frac{\frac{19}{29}}{\frac{4}{29}}\right)=\arctan\left(\frac{19}{4}\right)=\boxed{\mathbf{1.363}}$$

$$luego\,\frac{z_1}{z_4}=\boxed{\frac{\sqrt{\mathbf{377}}}{\mathbf{29}}(\mathbf{cos}(\mathbf{1.363})+\mathbf{sin}(\mathbf{1.363})\,\boldsymbol{i})}$$

c) Halle el valor de z en la siguiente igualdad

$$(2+3i)z+2i=3-4i$$

Metodo 1

$$(2+3i)z=3-6i$$

$$z=\frac{3-6i}{2+3i}\cdot\frac{2-3i}{2-3i}=\frac{(3-6i)(2-3i)}{4+9}=\frac{6-9i-12i+18i^2}{13}=$$

$$\frac{6-21i-18}{13}=\boxed{-\frac{\mathbf{12}}{\mathbf{13}}-\frac{\mathbf{21}}{\mathbf{13}}\boldsymbol{i}}$$